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1995 J. Phys. A: Math. Gen. 28 5581

(http://iopscience.iop.org/0305-4470/28/19/013)

J. Phys. A: Math. Gen. 28 (1995) 5581–5587. Printed in the UK

On the phase of Chern–Simons theory with complex gauge group

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Received 4 May 1995

Abstract. We compute the eta function for Chern–Simons quantum field theory with complex gauge group. The calculation is performed using the Schwinger expansion technique. We discuss, in particular, the role of the metric on the field configuration space, and demonstrate that for a certain class of acceptable metrics the one-loop phase contribution to the effective action can be calculated explicitly. The result is found to be proportional to a gauge-invariant part of the action.

1. Introduction

The effective action in certain three-dimensional topological field theories has been studied in [1–7]. Particular attention has been paid to the Chern–Simons theory [8], and its BF-type counterparts [9–11], when the gauge group is a compact non-Abelian group. The relevance of these calculations lies in the fact that as one is dealing with a topological field theory, the nature of all quantum corrections must be identified explicitly. While one may naively have a metric independent (topological) action at the classical level, one must ensure the preservation of these special properties by demonstrating that possible quantum corrections to the effective action maintain the topological nature of the model. Therefore, it is important to examine in full generality the nature of quantum corrections and, to this end, the off-shell effective action was studied in the above references.

The general formalism adopted there was the Vilkovisky–DeWitt [12, 13] off-shell effective action formalism, which provides the necessary geometrical framework in which to study the relevant quantum corrections. As shown in [3], the application of this formalism to the first-order theories revealed some interesting subtleties associated with the path integral measure. In particular, we mention the fact that the $\sqrt{G_{ij}}$ factor in the functional integral measure plays a crucial role in ensuring a successful application of the VD formalism, where G_{ij} denotes the metric on the field configuration space. While this factor can normally be overlooked in second-order theories, when the field metric is field independent, this is not the case in first-order models. In the latter, this metric factor alters in an essential way the phase contribution (the eta function) to the one-loop effective action.

In [14], the quantization of Chern–Simons theory for the case of a complex gauge group was studied. An appealing feature of this model is that it describes 2 + 1 quantum gravity, when the gauge group is chosen to be $SL(2, C)$. Indeed, both the Lorentzian and Euclidean signatures cases are dealt with in this scenario. As discussed above, it is therefore of general interest to examine the nature of the quantum corrections to this model, in particular we would like to extract the general off-shell behaviour. To this end, we

0305-4470/95/195581+07$19.50

concentrate our analysis here on the evaluation of the effective action to one-loop order. At first sight, this model bears a formal resemblance to a coupled system containing an interaction between a Chern–Simons and a BF model. However, on closer inspection, one notices that certain relative signs in the interaction terms differ from those in the coupled model just mentioned. In the construction of the effective action, one then finds that the metric on the field configuration space enjoys a different structure. Since the construction of an acceptable metric on the field configuration space is at the heart of the geometric approach of Vilkovisky–DeWitt [12, 13], this difference has immediate consequences.

Our strategy in the sequel is to adopt the formalism of Vilkovisky–DeWitt, in which the gauge fixing independent effective action is obtained. As is well established, the primary object in this analysis is the identification of an acceptable metric on the field configuration space. We shall first obtain the most general acceptable metric in this context, and then show that an explicit computation of the phase contribution to the one-loop off-shell effective can be performed for a particular family of metrics. The restriction to a particular class of metrics is needed in order to allow a complete resolution of the expansion terms in the Schwinger expansion. The net result is that one finds the eta function is proportional only to a part of the original classical action. However, the important observation is that the action of Chern–Simons theory with complex gauge group is the sum of two parts, each of which is independently gauge invariant. With our particular choice of field metric, we find an eta function proportional to one of these gauge invariant parts.

2. Chern–Simons theory with complex gauge group

In [14], the structure of Chern–Simons quantum field theory for the case of a complex gauge group was studied. Let us begin by presenting the classical action of this model:

$$S_c = \tfrac{1}{2}k \int d^3x \ \mathrm{tr}\,\epsilon^{\alpha\beta\gamma}(A_\alpha \partial_\beta A_\gamma + \tfrac{2}{3}A_\alpha A_\beta A_\gamma - B_\alpha D_\beta B_\gamma)$$
$$-k' \int d^3x \ \mathrm{tr}\,\epsilon^{\alpha\beta\gamma}(\tfrac{1}{2}B_\alpha F_{\beta\gamma} - \tfrac{1}{3}B_\alpha B_\beta B_\gamma) = kS_1 - k'S_2. \qquad (1)$$

Here, A_α is the gauge field with curvature $F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha + [A_\alpha, A_\beta]$, while $B = B^a_\alpha T^a dx^\alpha$ is a 1-form in the adjoint representation of the gauge group, and k and k' are two coupling parameters in the model. The covariant derivative is defined by $D_\alpha B_\beta = \partial_\alpha B_\beta + [A_\alpha, B_\beta]$. Implicit in our formulae is that the trace is taken in the fundamental representation. We present our analysis for the theory defined on flat space $\mathbb{R}^3$, so that the momentum space procedure employed is valid. Conventionally, we take the structure constants to be real and completely antisymmetric, with $[T^a, T^b] = f^{abc}T^c$. For the fundamental representation of $SU(n)$, the matrices T^a are skew-Hermitian and we take $\mathrm{tr}\, T^a T^b = -\frac{1}{2}\delta^{ab}$, while for the quadratic Casimir we have $f^{acd} f^{bcd} = c_v \delta^{ab}$.

We shall be concerned with the partition function in the following form:

$$Z = \int dA\, dB \exp\left(\frac{ik}{2\pi}(S_1 - rS_2)\right) \qquad (2)$$

where we have introduced the ratio of couplings $r = k'/k$.

Notice that we have identified two basic parts of the total action, S_1 and S_2; one can readily check that they are independently invariant under the gauge transformations to be presented. At this point, one can see a resemblance with other topological field theories, most notably Chern–Simons gauge theory and BF theory with compact gauge groups. The general coupling of these models, and their one-loop structure, pertinent to the present discussion, was investigated in various works [1–7]. However, it should be pointed out that the complex nature of the gauge group under study here has the effect of altering certain signs in the interaction terms. It transpires that these relative sign changes have a significant effect on the relevant metric of the field configuration space, which we treat in the following.

The local symmetries of (1) are easily established, and are given by

$$\delta A_\alpha = D_\alpha \omega - [B_\alpha, \theta] \qquad \delta B_\alpha = D_\alpha \theta + [B_\alpha, \omega]. \tag{3}$$

We first split the fields into a quantum part plus a background part, as the first step in a one-loop background field calculation, as follows

$$A \to A + A^q \qquad B \to B + B^q. \tag{4}$$

With this decomposition, the symmetries are:

$$\begin{aligned} \delta A_\alpha &= D_\alpha \omega - [B_\alpha, \theta] \qquad & \delta A^q_\alpha &= [A^q_\alpha, \omega] - [B^q_\alpha, \theta] \\ \delta B_\alpha &= D_\alpha \theta + [B_\alpha, \omega] \qquad & \delta B^q_\alpha &= [B^q_\alpha, \omega] + [A^q_\alpha, \theta] \end{aligned} \tag{5}$$

where the covariant derivative is with respect to the background field. We now need to quantize the theory. To carry out this quantization, one needs to look for a set of gauge fixing conditions which are covariant with respect to the symmetries (5). One can check that the following set satisfies this condition:

$$G_\phi \equiv DA^q - [B_\alpha, B^{q\alpha}] = 0 \qquad G_\pi = DB^q + [B_\alpha, A^{q\alpha}] = 0 \tag{6}$$

which indeed transform appropriately as follows:

$$\delta G_\phi = [G_\phi, \omega] - [G_\pi, \theta] \qquad \delta G_\pi = [G_\pi, \omega] + [G_\phi, \theta]. \tag{7}$$

By implementing these covariant gauge fixing conditions in the quantum action, and considering the part which is quadratic in the quantum fields, one obtains:

$$\begin{aligned} \frac{1}{k} S^{(2)}_q = \int \mathrm{d}^3 x \ \mathrm{tr}\{ & \epsilon^{\alpha\beta\gamma} \tfrac{1}{2}(A^q_\alpha D_\beta A^q_\gamma - B_\alpha D_\beta B_\gamma - 2B_\alpha [A^q_\beta, B^q_\gamma] \\ & - r(2B^q_\alpha D_\beta A^q_\gamma + B_\alpha [A^q_\beta, A^q_\gamma] - B_\alpha [B^q_\beta, B^q_\gamma])) \\ & + \phi(G_\phi - \tfrac{1}{2}\alpha\phi) + \pi(G_\pi - \tfrac{1}{2}\alpha'\pi)\} + \text{ghosts} \end{aligned} \tag{8}$$

where one recalls that the ghost and multiplier fields do not possess classical backgrounds. Here, α and α' are two arbitrary gauge fixing parameters. For the purposes of our computations, we shall adopt the gauge in which both of these parameters are set to zero, this is the gauge in which the Vilkovisky–DeWitt correction terms vanish, as can be checked. The result of our calculation in this gauge will therefore represent the unique value of the effective action (for the particular chosen field space metric).

We are interested in the first-order matrix $H^{\alpha\beta}_{ab}$ connecting the gauge fields and multipliers. It is given by:

$$\begin{aligned} S^{(2)}_q = \tfrac{1}{4} \int \mathrm{d}^3 x \, (B^q_\alpha A^q_\alpha \phi \pi)^a & \\ \times \begin{pmatrix} \epsilon^{\alpha\gamma\beta}(D^{ab}_\gamma - r f^{acb} B^c_\gamma) & \epsilon^{\alpha\gamma\beta}(r D^{ab}_\gamma + f^{acb} B^c_\gamma) & -R f^{acb} B^c_\alpha & S D^{ab}_\alpha \\ \epsilon^{\alpha\gamma\beta}(r D^{ab}_\gamma + f^{acb} B^c_\gamma) & \epsilon^{\alpha\gamma\beta}(-D^{ab}_\gamma + r f^{acb} B^c_\gamma) & R D^{ab}_\alpha & S f^{acb} B^c_\alpha \\ R f^{acb} B^c_\beta & -R D^{ab}_\beta & 0 & 0 \\ -S D^{ab}_\beta & -S f^{acb} B^c_\beta & 0 & 0 \end{pmatrix} & \\ \times \begin{pmatrix} B^q_\beta \\ A^q_\beta \\ \phi \\ \pi \end{pmatrix}^b . & \end{aligned} \tag{9}$$

Equation (9) defines a rank-2 symmetric object H_{ij}, which lies between the fields Φ^i and Φ^j, where we adopt the collective notation $\Phi^i \equiv (B^{qa}_\alpha(x), A^{qa}_\alpha(x), \phi^a(x), \pi^a(x))$. Also notice that the multiplier fields ϕ and π have been scaled by R and S respectively. This shall offer a convenience at a later stage in our calculation.

The theory is defined via the partition function

$$Z = \int \sqrt{\det G_{ij}} \prod_i \mathrm{d}\Phi^i \, \mathrm{e}^{\mathrm{i}\Phi^i H_{ij} \Phi^j} \tag{10}$$

the result of which is

$$\det{}^{1/2}[G_{ij}] \det{}^{-1/2}[H_{ij}] = \det{}^{-1/2}[H^i_j]. \tag{11}$$

Even when the metric G_{ij} is field independent, its presence in (11) as $\sqrt{\det G_{ij}}$ plays a crucial role in ensuring the field reparametrization invariance of the path integral [15]. This has also been amply shown in obtaining the correct eta function contribution to various topological field theories, [3, 6, 7]. Therefore, one must address the issue of which field space metric is appropriate. We proceed by re-writing the transformations (3) in the condensed notation as follows:

$$\delta\phi^i = K^i_\alpha \epsilon^\alpha \tag{12}$$

where the field ϕ^i labels only the classical fields A and B. The symmetry generators are denoted by K^i_α, and the infinitesimal gauge parameters are $\epsilon^\alpha = (\omega, \theta)$. An acceptable field metric, G_{ij}, is defined by the requirement that it admits the gauge generators, K^i_α, as Killing vectors [12, 13]. In other words, the following relation between the acceptable G_{ij} and K^i_α must hold:

$$0 = G_{ik}\partial_j K^k_\alpha + G_{jk}\partial_i K^k_\alpha + (\partial_k G_{ij}) K^k_\alpha \tag{13}$$

for all α. In this particular case, one can establish that a constant, field independent, metric which satisfies (13) does exist. It is shown to be of the form:

$$G_{ij} = \begin{pmatrix} \sigma & \lambda \\ \lambda & -\sigma \end{pmatrix} \delta^{ab}_{\alpha\beta}\delta(x-y) \tag{14}$$

with $\sigma^2 + \lambda^2 \neq 0$. The point to note here is the difference between the above acceptable metric, and that involved in the coupling of Chern–Simons and BF theory in which the G_{AA} component is equal to the G_{BB} component [6].

We must now obtain a suitable metric on the multiplier space. Based on the geometrical arguments, the multiplier metric is obtained by requiring the invariance of $G_{\alpha\beta}\lambda^\alpha\lambda^\beta$, where λ^α labels the multiplier fields ϕ and π, that is $\lambda^\alpha = (\phi, \pi)$. Since we already have the transformations of the gauge fixing conditions, we need to obtain those of the multiplier fields. Under the requirement that $\phi G_\phi + \pi G_\pi$ be invariant with respect to (5), we find that the multiplier fields transform as:

$$\delta\phi = [\phi, \omega] + [\pi, \theta] \qquad \delta\pi = [\pi, \omega] - [\phi, \theta]. \tag{15}$$

Given these transformations, we can now determine the multiplier metric by requiring:

$$\delta(G_{\alpha\beta}\lambda^\alpha\lambda^\beta) = 0. \tag{16}$$

The most general solution of (16) is found to be of the form:

$$G_{\alpha\beta} = \begin{pmatrix} \sigma' & \lambda' \\ \lambda' & -\sigma' \end{pmatrix} \delta^{ab}_{\alpha\beta}\delta(x-y) \tag{17}$$

where $\sigma'^2 + \lambda'^2 \neq 0$. Therefore, under these conditions, the most general solution of the full metric on the gauge field-multiplier space, G_{ij}, is given by:

$$G_{ij} = \begin{pmatrix} \sigma\delta_{\alpha\beta} & \lambda\delta_{\alpha\beta} & 0 & 0 \\ \lambda\delta_{\alpha\beta} & -\sigma\delta_{\alpha\beta} & 0 & 0 \\ 0 & 0 & \sigma' & \lambda' \\ 0 & 0 & \lambda' & -\sigma' \end{pmatrix} \delta^{ab}\delta(x-y). \tag{18}$$

Assuming that λ and λ' are non-zero, we can rescale A^q and B^q by $1/\sqrt{\lambda}$, and π and ϕ by $1/\sqrt{\lambda'}$, to find that the inverse metric takes the form:

$$G^{ij} = \begin{pmatrix} \frac{u}{(1+u^2)}\delta_{\alpha\beta} & \frac{1}{(1+u^2)}\delta_{\alpha\beta} & 0 & 0 \\ \frac{1}{(1+u^2)}\delta_{\alpha\beta} & -\frac{u}{(1+u^2)}\delta_{\alpha\beta} & 0 & 0 \\ 0 & 0 & \frac{v}{(1+v^2)} & \frac{1}{(1+v^2)} \\ 0 & 0 & \frac{1}{(1+v^2)} & -\frac{v}{(1+v^2)} \end{pmatrix} \delta^{ab}\delta(x-y) \tag{19}$$

with $u = \sigma/\lambda$ and $v = \sigma'/\lambda'$.

Having dealt with the construction of the most general field metric, we can now proceed with the regularization of the path integral. We define the ζ and η functions of the operator $H \equiv H^i_j = G^{ik}H_{kj}$ via its eigenvalues λ_n:

$$\zeta_H(s) = \sum_n |\lambda_n|^{-s} \qquad \eta_H(s) = \sum_n (\mathrm{sign}\lambda_n)|\lambda_n|^{-s}. \tag{20}$$

This leads to the result that

$$Z_{\mathrm{reg}} = \mathrm{e}^{\frac{1}{2}\zeta'_H(0)+(\mathrm{i}\pi/4)\eta_H(0)}. \tag{21}$$

The η-function has the following integral representation:

$$\eta_H(s) = \frac{1}{\Gamma((s+1)/2)}\int_0^\infty \mathrm{d}t\, t^{(s-1)/2}\,\mathrm{Tr}[H\mathrm{e}^{-H^2 t}]. \tag{22}$$

In general, $\eta(s)$ is difficult to calculate for arbitrary values of s, but its value at $s = 0$ has a more manageable representation. To see this, one decomposes H as $H_0 + H_1$, where H_0 is independent of the background fields, and H_1 contains the interaction terms. One can then employ a trick due to Gilkey [16], see for example [1], to show that

$$\eta_H(0) = \lim_{s\to 0}\frac{-s}{\Gamma((s+1)/2)}\int_0^\infty \mathrm{d}t\, t^{(s-1)/2}[\mathrm{Tr}_0 + \mathrm{Tr}_1 + \mathrm{Tr}_2 + \cdots] \tag{23}$$

where,

$$\mathrm{Tr}_0 = \mathrm{Tr}[H_1\mathrm{e}^{-H_0^2 t}]$$
$$\mathrm{Tr}_1 = -t\int_0^1 \mathrm{d}u\,\mathrm{Tr}[\mathrm{e}^{-H_0^2 ut}H_1\mathrm{e}^{-H_0^2(1-u)t}(\tfrac{1}{2}\{H_0, H_1\} + \tfrac{1}{3}H_1^2)]$$
$$\mathrm{Tr}_2 = \tfrac{1}{3}t^2\int_0^1 u\,\mathrm{d}u\int_0^1 \mathrm{d}v\,\mathrm{Tr}[\mathrm{e}^{-H_0^2 uvt}H_1\mathrm{e}^{-H_0^2(1-u)t}\{H_0, H_1\}\mathrm{e}^{-H_0^2 u(1-v)t}\{H_0, H_1\}]. \tag{24}$$

The calculation then amounts to an evaluation of those terms which contribute a pole in s, so as to cancel the explicit s factor in (23); these can be performed most readily in momentum space with the definition

$$\mathrm{Tr}\,\mathcal{O} = \int \frac{\mathrm{d}^n p}{(2\pi)^n}\langle p|\mathrm{Tr}'\,\mathcal{O}|p\rangle = \int \frac{\mathrm{d}^n p;}{(2\pi)^n}\mathrm{d}^n x\,\mathrm{d}^n y\langle p|x\rangle\langle x|\mathrm{Tr}'\,\mathcal{O}|y\rangle\langle y|p\rangle. \tag{25}$$

Here, the prime indicates a trace over any other indices carried by the operator $\mathcal{O}$. Our conventions are such that $\langle p|x\rangle = \mathrm{e}^{-\mathrm{i}px}$ and $A(p) = \int \mathrm{d}^n x\mathrm{e}^{-\mathrm{i}px}A(x)$.

3. Computation of the eta function

Given the structures derived in the previous section, we can now present the details of the computation of the eta function.

The operator of interest is given by $H \equiv (G^{ik}H_{kj})^{ab}_{\alpha\beta}(x,y)$. As can be seen from the integral representation just presented, an important calculational simplicity is achieved when the free part of the H operator squares to a diagonal operator. This allows a complete resolution of the relevant contributions to the eta function.

In momentum space, we have the following representation for the free (H_0) part of H:

$$\langle p|H_0|q\rangle^{\alpha\beta}_{ab} = \mathrm{i}\delta_{ab}(p-q)\frac{1}{4}\begin{pmatrix} \frac{(u+r)}{(1+u^2)}\epsilon^{\alpha\gamma\beta}p_\gamma & \frac{(ur-1)}{(1+u^2)}\epsilon^{\alpha\gamma\beta}p_\gamma & \frac{R}{(1+u^2)}p_\alpha & \frac{uS}{(1+u^2)}p_\alpha \\ \frac{(1-ru)}{(1+u^2)}\epsilon^{\alpha\gamma\beta}p_\gamma & \frac{(r+u)}{(1+u^2)}\epsilon^{\alpha\gamma\beta}p_\gamma & -\frac{uR}{(1+u^2)}p_\alpha & \frac{S}{(1+u^2)}p_\alpha \\ -\frac{S}{(1+v^2)}p_\beta & -\frac{vR}{(1+v^2)}p_\beta & 0 & 0 \\ \frac{vS}{(1+v^2)}p_\beta & -\frac{r}{(1+v^2)}p_\beta & 0 & 0 \end{pmatrix}. \quad (26)$$

We now look for values of u, v, R and S such that H_0^2 becomes diagonal. Upon investigation, one finds that the following set renders this the case:

$$u = \frac{1}{r} \qquad v = -\frac{1}{r} \qquad R = S \qquad R^2 = (1+r^2). \quad (27)$$

The set (27) leads to:

$$\langle p|H_0^2|q\rangle^{\alpha\beta}_{ab} = \tfrac{1}{16}r^2\delta_{ab}(p-q)\begin{pmatrix} \delta^{\alpha\beta}p^2 & 0 & 0 & 0 \\ 0 & \delta^{\alpha\beta}p^2 & 0 & 0 \\ 0 & 0 & p^2 & 0 \\ 0 & 0 & 0 & p^2 \end{pmatrix}. \quad (28)$$

In addition, one then has the following representation for the interacting, H_1, part of H:

$$\langle p|H_1|q\rangle^{\alpha\beta}_{ab} = \tfrac{1}{4}f^{acb}M \quad (29)$$

where the elements of the matrix M are given by:

$$\begin{aligned}
&(1,1) = r\epsilon^{\alpha\gamma\beta}A^c_\gamma && (3,1) = -r(1+r^2)^{-1/2}(rA^c_\beta + B^c_\beta) \\
&(1,2) = r\epsilon^{\alpha\gamma\beta}B^c_\gamma && (3,2) = r(1+r^2)^{-1/2}(A^c_\beta - rB^c_\beta) \\
&(1,3) = r(1+r^2)^{-1/2}(rA^c_\alpha - B^c_\alpha) && (3,3) = 0 \\
&(1,4) = r(1+r^2)^{-1/2}(A^c_\alpha + rB^c_\alpha) && (3,4) = 0 \\
&(2,1) = -r\epsilon^{\alpha\gamma\beta}B^c_\gamma && (4,1) = -r(1+r^2)^{-1/2}(A^c_\beta - rB^c_\beta) \\
&(2,2) = r\epsilon^{\alpha\gamma\beta}A^c_\gamma && (4,2) = -r(1+r^2)^{-1/2}(rA^c_\beta + B^c_\beta) \\
&(2,3) = -r(1+r^2)^{-1/2}(A^c_\alpha + rB^c_\alpha) && (4,3) = 0 \\
&(2,4) = r(1+r^2)^{-1/2}(rA^c_\alpha - B^c_\alpha) && (4,4) = 0
\end{aligned} \quad (30)$$

where A above implicitly denotes $A(p-q)$.

We can now proceed with the calculation of the phase of the partition function. We find that the Tr_0 and Tr_2 do not contribute to $\eta_H(0)$, while the remaining terms give

$$\mathrm{Tr}[\mathrm{e}^{-H_0^2ut}H_1\mathrm{e}^{-H_2(1-u)t}\{H_0,H_1\}] \to -\frac{2c_v}{\pi^2}\int \mathrm{d}^3x\,\epsilon^{\alpha\gamma\beta}\,\mathrm{tr}_F(A_\alpha\partial_\gamma A_\beta - B_\alpha\partial_\gamma A_\beta) \quad (31)$$

and

$$\mathrm{Tr}[\mathrm{e}^{-H_0^2ut}H_1\mathrm{e}^{-H_0^2(1-ut)}H_1^2] \to -\frac{2c_v}{\pi^2}\int \mathrm{d}^3x\,\epsilon^{\alpha\gamma\beta}\,\mathrm{tr}_f(\tfrac{2}{3}A_\alpha A_\gamma A_\beta - B_\alpha[A_\gamma,B_\beta]) \quad (32)$$

where we have scaled t by $\frac{1}{16}t$. Combining these results (and noting that $\eta_{H(0)}(0) = 0$) we find that the complete expression for $\eta_H(0)$ is

$$\eta_H(0) = -\frac{2c_v}{\pi^2}\int \mathrm{d}^3x\, \epsilon^{\alpha\gamma\beta}\,\mathrm{tr}_F(A_\alpha\partial_\gamma A_\beta + \tfrac{2}{3}A_\alpha A_\gamma A_\beta - B_\alpha D_\gamma B_\beta). \tag{33}$$

As we can see, this result is proportional to the S_1 part of the action (1). Of course, the entire construction of the effective action just presented is based upon gauge invariant input, and *a priori* the value of the eta function derived in such a context should be gauge invariant.

4. Conclusion

We have explicitly constructed the phase contribution to the off-shell effective action, at one-loop order, for Chern–Simons gauge theory with complex gauge group. The important observation in this particular calculation is that the most general acceptable metric on the gauge field-multiplier configuration space assumes a form with crucial sign differences from that of a coupled theory with compact gauge group. Nevertheless, we succeeded in finding a class of metrics which allowed the calculation to be completely resolved. The result of the computation is that the phase, for this particular metric class, is proportional to only a gauge invariant part of the action. Such a result can be contrasted with Chern–Simons theory for a compact gauge group, where of course the eta function is proportional to the complete action itself.

Acknowledgments

SM is supported by the National Science Foundation under grant number PHY-9213090.

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